

two curves approach the center line (and hence the difference which will be obtained in the eddy viscosities) is readily seen. The two profiles differ at most by only 0.05%. Yet the center-line value of the eddy viscosity determined for the dashed curve, which comes in almost tangent to the center line, is approximately zero, whereas the center-line value for the solid curves is about 50% of the maximum value of the eddy viscosity for this profile.

The data of Nikuradse have been used here to illustrate the effect of a pointed velocity profile on the determination of the center-line value of the eddy viscosity. The data of other investigators could also have been used. In the work of Sage *et al.* (1, 2, 4) no tendency of the eddy viscosity to go to zero at the center line was reported. Values of the eddy viscosity were not given in the paper by Senecal and Rothfus (7), in which the region below Re 4,000 was studied. However Professor Rothfus (5) has informed the writer that low values of the eddy viscosity at the center line were obtained with Senecal's data used.

It was shown above that the low value of the eddy viscosity at the center line reported by Nikuradse on the basis of his data at 23,300 was obtained because the velocity profile was drawn without regard to its slope at the center line. The central core of a fluid flowing in a tube or channel is a zone of relatively uniform, nearly isotropic turbulence. A velocity profile drawn through a set of experimental points in this region should show no sharp changes in the first or second derivatives unless the data demand it, and on the basis of symmetry the first derivative at the center line must be zero. In other words a velocity profile should be no sharper than the scatter of the data requires. The method of plotting illustrated in Figure 2 is a

convenience in meeting these requirements.

The authors depended primarily on three sources for their eddy viscosities (1, 3, 7). Using the above criteria in plotting the writer has found that the data of Nikuradse (3) and Sage *et al.* (1) show a center-line value of the eddy viscosity which is generally 50% of the maximum value in the channel or greater. The same thing can probably be said of the data of Senecal and Rothfus. Such a relatively high value of the eddy viscosity is in accord with the work on eddy conductivity by Sage *et al.* (4) and is supported by the general picture of turbulent transport.

LITERATURE CITED

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Reply

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It is surely possible that the eddy-viscosity pattern close to the center of a tube may be quite different from that shown in the authors' original paper. There are certain difficulties with the experimental velocity data, particularly with the center-line values. On the other hand it is hard to say whether these data should be set aside in favor of results on other modes of transfer or extrapolations based on certain models of the transport process. The central region is very complicated because small gradients of transferable properties may be important there, even though they are insignificant elsewhere in the stream. In addition the degree of correlation among the components of the velocity fluctuations changes rapidly with position in the vicinity of the center, making it hard to predict what may happen in the limit as the center is approached. All in all it seems that the eddy-viscosity profile near the center is largely uncertain and that the nature of the turbulent transport in this region remains the thing to be determined by further work.

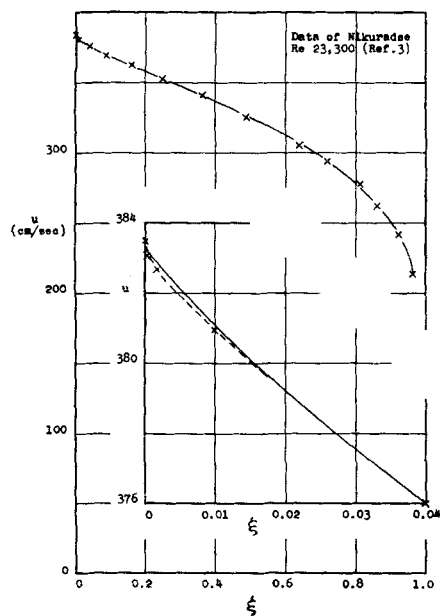


Fig. 2. Velocity vs. square of radial position.